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ANALYTICAL AND EXPERIMENTAL INVESTIGATION ON VIBRATING MEMBRANES WITH A CENTRAL POINT SUPPORT

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1. INTRODUCTION

In a recent, interesting study Wang [1] analyzed transverse vibrations of polygonal membranes with a central, fixed circular core by means of the point matching technique. He also studied in detail the asymptotic properties where the center core shrinks to a point, i.e., the membrane is point-supported at the center. Furthermore, Wang's study shows that in the case of the circular, annular membrane with a pinpoint restraint ($\varepsilon = 0$) one obtains the fundamental frequency of the solid circular membrane. Accordingly he expects this property to hold for polygonal membranes [1].

It was shown in a recent discussion that the presence of a pinpoint support at the center of a circular membrane does not alter the values of higher frequency coefficients corresponding to axisymmetric modes [2].

The present study deals with an experimental study on vibrating circular and square membranes with a central, point support complementing reference [2] by including additional numerical information on higher axisymmetric modes of a circular membrane.

2. THE VIBRATING CIRCULAR MEMBRANE WITH A CENTRAL PINPOINT SUPPORT

The classical problem of vibrating membranes is described, in the case of normal modes, by the Helmholtz equation,

$$\nabla^2 \Psi + k^2 \Psi = 0, \tag{1}$$

where k is the eigenvalue corresponding to a normal mode.

When dealing with a vibrating circular annular membrane described by the axisymmetric modes equation (1) and the prescribed boundary conditions at $r = b$, a, where b is the outer radius and a is the inner radius of the membrane, the solution yields the well known frequency equation

$$Y_0(k\,b)\,J_0(k\,a) - J_0(k\,b)\,Y_0(k\,a) = 0 \tag{2}$$

Defining now

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$$\varepsilon = a/b \tag{3}$$

and substituting in equation (2) one obtains

$$Y_0(\Omega)J_0(\Omega\varepsilon) - J_0(\Omega)Y_0(\Omega\varepsilon) = 0, \tag{4}$$

where $\Omega = kb$ and where the usual notation for Bessel's function of first and second kind has been used.

As ε becomes very small in equation (4) one is able to approach, asymptotically, the case of a central, pinpoint support [2].

Values of $\Omega_i = \sqrt{\rho/S}\,\omega_i b$ (where ρ is the density per unit area of the membrane and S is the applied force per unit length applied at each boundary of the membrane) are shown in Table 1 for the first five modes of vibration. Also shown in the table are the exact eigenvalues corresponding to axisymmetric modes of vibration of a solid circular membrane.

The numerical determinations have been performed using MAPLE [3]. It is concluded that as ε decreases and reaches the value 10^{-1000} the differences with the exact eigenvalues corresponding to a solid, circular membrane are of the order of 0·01%. This conclusion agrees with the one reached in reference [3] for the first three modes.

3. EXPERIMENTAL INVESTIGATION

Figure 1 depicts the experimental set-up used in the case of a vibrating circular membrane.* The first three natural frequencies corresponding to axisymmetric modes were excited using a commercial speaker. A specially developed infrared transducer [4] was used to detect the frequencies which were analyzed by means

TABLE 1

The first five frequency coefficients of a circular annular membrane as a function of $\varepsilon = a/b$ and as ε becomes very small

ε	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
10^{-1}	3·3139	6·8575	10·3774	13·8864	17·3896
10^{-2}	2·8009	6·0109	9·2141	12·4113	15·6043
10^{-3}	2·6548	5·8089	8·9676	12·1251	15·2811
10^{-4}	2·5871	5·7236	8·8698	12·0170	15·1641
10^{-5}	2·5482	5.6769	8·8181	11·9614	15.1052
10^{-10}	2·4741	5·5929	8·7283	11·8673	15·0075
10^{-100}	2·4115	5·5269	8·6605	11·7984	14·9378
10^{-1000}	2·4055	5·5207	8·6544	11·7922	14·9316
Exact	2·4048	5·5200	8·6537	11·7915	14·9309

Note: the values defined as "exact" correspond to axisymmetric modes of a solid, circular membrane.

*A professional Brazilian musical drum was used ($2b$ = 150 mm).

Figure 1. Experimental set-up and test of the circular membrane.

of CSI 1900. Experiments were performed with and without a central, point support. When performing the first set of experiments, special care was placed on not introducing variations of the in-plane stress field corresponding to the membrane without the central support.

Typical experimental results are shown in Table 2. It is concluded that placing a pinpoint central support raises the fundamental frequency by about 7%, which can be considered a very small increment in view of the inherent difficulties caused by testing a real membrane. (In the case of a simply supported circular plate with a central support the fundamental frequency is practically three times larger than the one corresponding to the plate without a central support [5].) The increment is considerably smaller in the case of the second and third modes; see, Table 2.

A square membrane was constructed out of thin leather of dimensions 128×128 mm and $1{\cdot}2$ mm thick and tested; see Figure 2. The experimental

TABLE 2

Values of natural frequencies determined experimentally in the case of a circular membrane: effect of the central, point support

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
Without central support	217·5	458·8	805·5
With central support	232·5	472	808

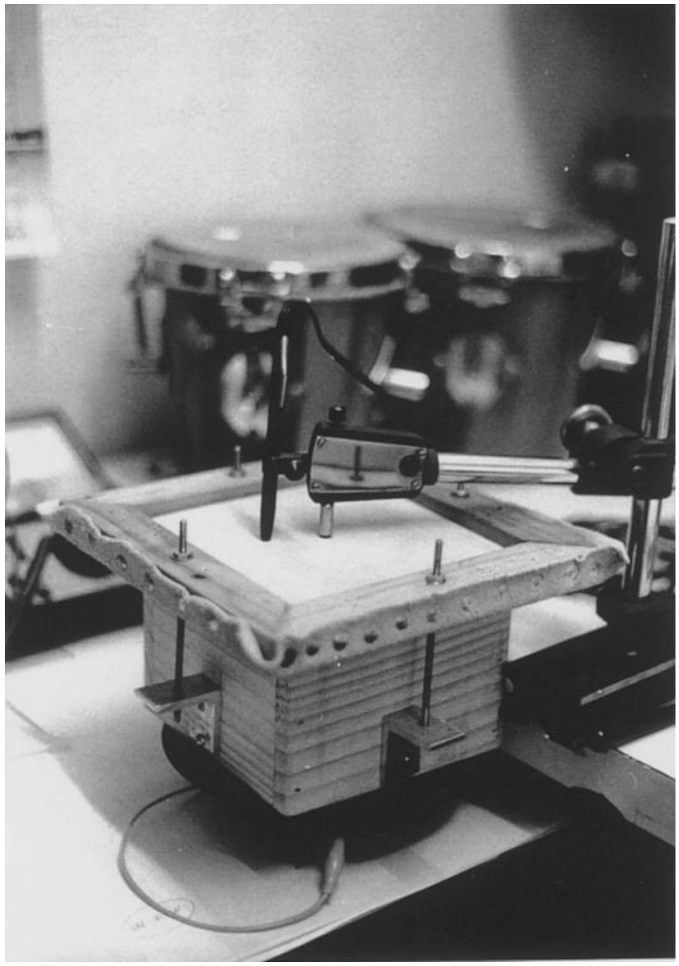

Figure 2. Experimental set-up and test of the square membrane.

procedure followed was similar to the one used when testing the circular membrane.

Table 3 shows the first three frequencies corresponding to symmetric modes when the membrane is centrally supported and when it is only boundary-supported. One concludes, again, that the frequency variation is negligible for

TABLE 3

Values of natural frequencies corresponding to doubly-symmetric modes in the case of a square membrane determined experimentally: effect of the central, point support

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
Without central support	149·9	315	750·9
With central support	175·4	318·5	749·7

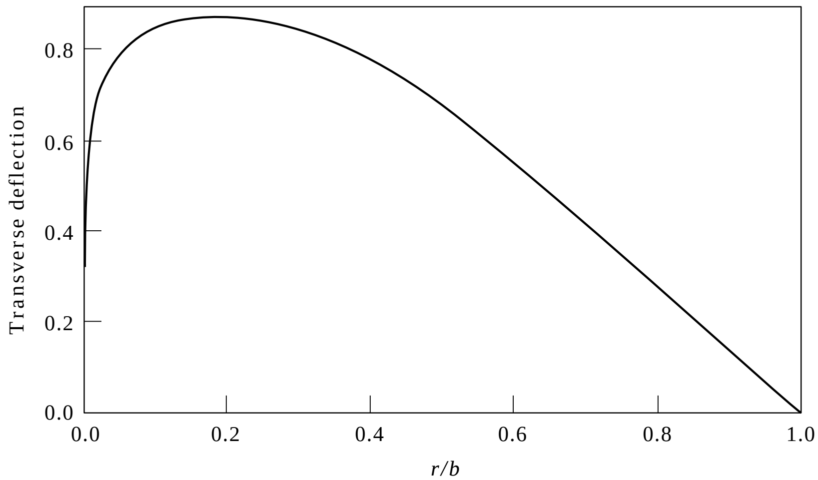

Figure 3. Fundamental mode shape of the annular membrane for $\varepsilon = 10^{-5}$.

the second and third frequencies and more noticeable in the case of the fundamental frequency.

Finally, Figures 3 and 4 depict the fundamental mode shapes for $\varepsilon = 10^{-5}$ and 10^{-100}, respectively, in the case of the circular annular membrane. It is observed that as ε becomes very small ($\varepsilon = 10^{-100}$), the mode shape resembles the functional relation defined by Bessel's function of the first kind and order zero, with a sudden drop at $\varepsilon = 10^{-100}$ in order to satisfy the prescribed boundary condition.

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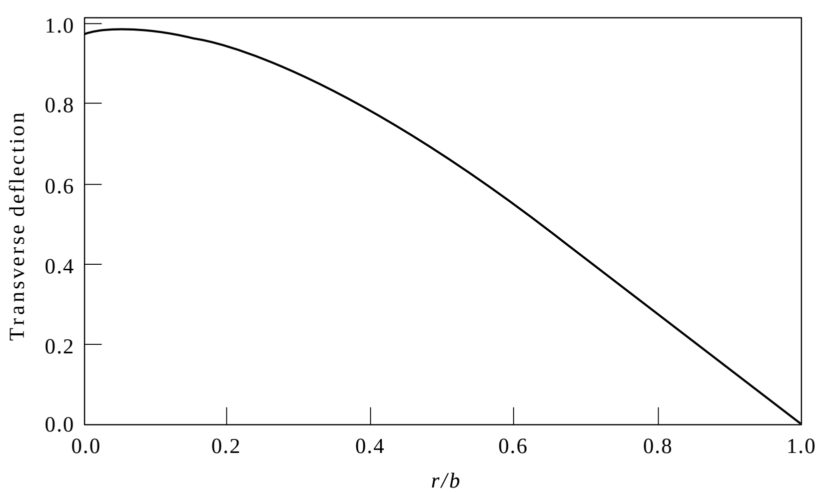

Figure 4. Fundamental mode shape of the annular membrane for $\varepsilon = 10^{-100}$.

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